Complexity Theory

Homework Sheet 4

Hand in before the lecture of Tuesday 7 Mar.

Preferably by email to bannink@cwi.nl

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Exercise 1. Show that $\mathbf{NL} \subseteq \mathbf{P}$.
Hint: Consider the configuration graph of the nondeterministic machine.

Exercise 2. In this exercise, all graphs are directed graphs. We define the following decision problem:

$$\textsc{deadend} = \{\langle G, s, t\rangle \mid \text{The graph } G \text{ contains a vertex } v, \text{ reachable from vertex } s, \text{ such that } t \text{ is not reachable from } v.\}$$

Show that this problem is contained in $\mathbf{NL}$.

Exercise 3. Define the complexity class

$$\mathbf{DP} = \{A \cap B \mid A \in \mathbf{NP}, B \in \mathbf{coNP}\}.$$

We say an undirected graph G has a *clique* of size k if there exists a subset S of k vertices such that all pairs of vertices in S have an edge between them.

$$\textsc{eclique} = \{\langle G, k\rangle \mid \text{the largest clique in the graph } G \text{ has exactly } k \text{ vertices}\}.$$

(a) Show that $\textsc{eclique} \in \Sigma_2^p \cap \Pi_2^p$.

(b) Show that $\textsc{eclique} \in \mathbf{DP}$.

(c) Show that if $\mathbf{DP} \subseteq \mathbf{NP}$, then the polynomial-time hierarchy collapses.

Exercise 4. Define $\mathbf{P}/log$ as the class of sets A for which there is an advice function $\alpha : \mathbb{N} \to \{0,1\}^{O(\log n)}$ and a polytime machine M such that for any string x of length *at most* n,

$$x \in A \iff M(x, \alpha(n)) = 1.$$

(a) Show that if $\textsc{sat}$ is in $\mathbf{P}/log$, then $\mathbf{P} = \mathbf{NP}$.

(b) **Bonus** - Show that $\mathbf{P} \subsetneq \mathbf{P}/log$; i.e. show that there is a decidable set in $\mathbf{P}/log$ that is not in $\mathbf{P}$.