Complexity Theory

Homework Sheet 3

Hand in before the lecture of Tuesday 28 Feb.

Preferably by email to bannink@cwi.nl

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Exercise 1. Is there an oracle such that, relative to this oracle, ...? If so, then exhibit such an oracle and prove it works. If not, prove why not.

(a) $\mathbf{P} = \mathbf{EXP}$

(b) $\mathbf{coNP} \subseteq \mathbf{P}$ *and* $\mathbf{NP} \not\subseteq \mathbf{P}$

(c) $\mathbf{NP} = \mathbf{coNP} \neq \mathbf{EXP}$

For example, in **(a)** you have to either show: there exists an oracle A such that $\mathbf{P}^A = \mathbf{EXP}^A$, or: such an oracle does not exist. In **(b)** you show either: there exists an oracle A such that $\mathbf{coNP}^A \subseteq \mathbf{P}^A$ and $\mathbf{NP}^A \not\subseteq \mathbf{P}^A$, or: such an oracle does not exist.

Exercise 2. Let $\mathbf{C}$ be the class of sets decidable by Turing machines which use polynomial space, but are not allowed to reuse space. I.e., the Turing machine can write over blank tape cells, but it can neither erase nor overwrite previously used cells – it *is* allowed to go back over the tape and read them.

(a) Prove that $\mathbf{C} \subseteq \mathbf{P}$.

(b) Prove that $\mathbf{P} \subseteq \mathbf{C}$.

Definition 1. We say that a set A is *1-query length-decreasing self-reducible* if there is a polytime oracle Turing machine M, such that

$$x \in A \iff M^A(x) = 1.$$

Moreover, the computation of $M^A(x)$ makes at most one query to the oracle, and that query has to be a string of length strictly less than $|x|$.

Exercise 3. Show that every 1-query length-decreasing self-reducible set is in $\mathbf{P}$.

Exercise 4. Prove that in the certificate definition of $\mathbf{NL}$ (Book §4.3.1), if we allow the verifier machine to move its head back and forth on the certificate, the class being defined changes to $\mathbf{NP}$. Hint: Consider 3-SAT.