Complexity Theory

Homework Sheet 4

(Turn in before the lecure of Monday 28 Apr.)

21 April, 2014

Exercise 1. Show that NL $\subseteq$ P.
Hint: Consider the configuration graph of the nondeterministic machine.

Exercise 2. We define the following decision problem:

$$\text{DEADEND} = \{\langle G, s, t\rangle \mid \text{The graph } G \text{ contains a vertex } v, \text{ reachable from vertex } s, \text{ such that } t \text{ is not reachable from } v.\}$$

Show that this problem is contained in NL.

Exercise 3. Define the complexity class

$$\text{DP} = \{A \cap B | A \in \text{NP}, B \in \text{coNP}\}.$$

We say a graph G has a *clique* of size k if there exists a subset S of k vertices such that all pairs of vertices in S have an edge between them.

$$\text{ECLIQUE} = \{\langle G, k\rangle \mid \text{the largest clique in the graph } G \text{ has exactly } k \text{ vertices}\}.$$

(a) Show that ECLIQUE $\in \Sigma_2^p \cap \Pi_2^p$.

(b) Show that ECLIQUE $\in$ DP.

(c) Show that if DP $\subseteq$ NP, then the polynomial-time hierarchy collapses.

(d) *(Bonus question for +1 point.)* Show that ECLIQUE is DP-complete, i.e., that every set in DP is $\leq_m^p$-reducible to ECLIQUE. Hint: look up the NP-completeness of the clique problem and combine the result of the reductions in a clever way.